GÜNTER M. ZIEGLER 60

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There are two cores in Günter's numerous and widely recognized scientific achievements. The first topic are convex polytopes. Günter's landmark textbook "Lectures on polytopes" from 1995 played a key role in putting the theory of convex polytopes on the map of mathematics as a field of its own right. Previously, polytopes were largely seen through the lens of optimization, where polytopes occur as feasible regions of linear programs.

Günter's second main topic is the rich interplay of combinatorics and topology, where the benefit is mutual. The stage has been set in his first paper, "On the poset of partitions of an integer", which appeared in the *Journal of Combinatorial Theory, Series A* in 1986, one year before he finished his PhD at MIT. The objects of interest are the posets $\mathcal{P}_n$, whose elements are the partitions of the integer n, partially ordered by refinement. Anders Björner, who was Günter's supervisor at MIT, had conjectured that $\mathcal{P}_n$ always satisfies the Cohen-Macaulay (CM) property, which is a subtle notion at the border between commutative algebra and topology; this was known to be true for $n \leq 8$. Recall that any partially ordered set naturally carries a topological structure, as the chains form an abstract simplicial complex. Günter proves that $\mathcal{P}_n$ is not CM for $n \geq 19$. The proof is fully explicit, there is a lot of extra context (including proofs that $\mathcal{P}_9$ and $\mathcal{P}_{10}$ are CM, another result on $\mathcal{P}_n$ for $n \geq 111$, an extension and a correction of a known result of Birkhoff from 1967), the text is easy to read, a slick counter-example refutes an intriguing conjecture. And the entire paper only has eight pages. The combination of these features characterizes Günter's mode of operation until today, with more than 170 published articles and books, and counting.

Now it is important to know that the boundary complex of a (simplicial) polytope is always shellable, a property which implies CM. In this way, the two cores of Günter's work become actually just one. So, "Lectures on polytopes" teaches us that linear optimization, topology, commutative algebra and combinatorics are best studied as a rich mix, rather than separate fields.

In the mid-90s I came to Berlin, as a postdoc, to work with and learn from Günter. This is when he inspired Ewgenij Gawrilow and me to start developing the software system `polymake`, which became a crucial step in my academic career. Thank you, Günter, and Happy Birthday!