Berlin Mathematics Research Center
MATH+

Florian Frick on John M. Sullivan's research

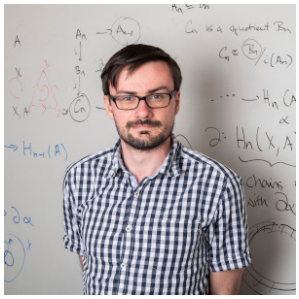
A geometric shape that minimizes a given energy functional can typically be expected to be symmetric or otherwise visually pleasing, such as a soap bubble that minimizes its area while enclosing a fixed volume. Such visual geometric optimization problems are central to John Sullivan's research work.

"How much rope does it take to tie a knot?" is the beginning of one of John Sullivan's papers, joint with Cantarella and Kusner, that contains improved bounds for this question. The same authors show that the minimum crossing number of a knot, when drawn in the plane, may grow superlinearly in the rope length, disproving an earlier conjecture that originated in biophysics. Subsequent work with Cantarella, Fu, Kusner, and Wrinkle extends methods developed for this rope length problem to multi-component links, including an intricate description of Borromean rings built from unit-diameter rope and pulled as tight as possible. This configuration is the basis of Sullivan's design of the current logo of the International Mathematical Union. Gromov showed that on any closed curve, there are two points whose distance along the curve is greater by a factor of at least pi/2 compared to the straight-line distance, as the crow flies. Together with his former student Elizabeth Denne, Sullivan showed that this distortion is at least 5pi/3 for a curve that is knotted.

Complementing his diverse work in geometric knot theory, another theme of Sullivan's research concerns foams and froths. In 1887 Lord Kelvin proposed a foam that he conjectured to be a minimum-area decomposition of space into cells of equal volume. A century later, Denis Weaire and Robert Phelan constructed a foam that beat Kelvin's decomposition by a tiny margin. Weaire-Phelan gave numerical evidence for this claim. A rigorous, analytical proof that the Weaire-Phelan foam is of smaller total area in each periodic building block was provided by Kusner and Sullivan. Mathematically a foam is a collection of constant mean curvature surfaces that meet according to Plateau's rule. Another landmark result of Sullivan (with Große-Brauckmann and Kusner) in the area includes a complete description of the moduli space of constant mean curvature pants, the basic building blocks of more complicated constant mean curvature surfaces.

Sullivan's mathematical work, often motivated by physics, chemistry, and the visually appealing, lends itself easily to his broader mission as an educator, an artist, and a communicator of mathematics. Drawing on a broad knowledge base and intuition that can only stem from decades of experience with the visual, John Sullivan has a propensity for asking the right questions -- an invaluable quality in an advisor.

Dear John, please accept our best wishes for your sixtieth birthday!



Florian Frick (© Carnegie Mellon University)

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