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Anna von Pippich on Jürg Kramer's Research

"There are five basic operations of arithmetic: addition, subtraction, multiplication, division, and modular forms." This is a quote by Martin Eichler (1912-1992), Jürg Kramer's PhD advisor at the University of Basel, who introduced him to the magic power of modular forms in number theory. A famous example for this power is the crucial role of modular forms in Andrew Wiles's proof of Fermat's Last Theorem in 1994, which states that there are no non-zero integral solutions x, y, z to the equation $x^n + y^n = z^n$ as soon as $n \geq 3$.

The theory of modular forms or, more generally, the theory of automorphic form is one main field of Jürg Kramer's research activity. While trigonometric functions are periodic functions on the real line, modular forms are periodic functions on the hyperbolic plane or on higher dimensional analogs. One main result obtained by Jürg Kramer, jointly with Jay Jorgenson, concerns the fundamental question of understanding bounds for automorphic forms. In particular, their work establishes optimal sup-norm bounds for the average value of any orthonormal basis of weight two automorphic forms. Strikingly, to achieve these arithmetic results, Jorgenson-Kramer combined methods from the underlying hyperbolic geometry of modular curves with heat kernel analysis. The second main field of Jürg Kramer's research activities is Arakelov theory, which has its origins in the pioneering work on arithmetic surfaces by Suren Arakelov in the 80s. In the early 90s, Gillet-Soulé extended Arakelov's work to higher dimensional arithmetic varieties. One important application of Arakelov theory is that the height of a point or of a higher dimensional cycle, a measure for the arithmetic complexity of that point or cycle, can be computed by means of arithmetic intersection theory. In joint work with José Burgos Gil and Ulf Kühn, Jürg Kramer was able to generalize the Arakelov theory of Gillet-Soulé to a certain logarithmically singular setting. This foundational work is crucial for future arithmetic applications, in particular, it allows the treatment of modular curves.

Another direction of research aims at establishing bounds for Arakelov invariants, such as the Faltings's delta function, that was introduced by Faltings as a measure for the distance to the boundary of the moduli space, for a complex Riemann surface. In an important work, Jorgenson-Kramer proved a bound for this function in terms of geometric data, such as the lengths of closed geodesics or the smallest eigenvalue of the hyperbolic Laplacian. The key idea of their proof consists in a translation from Arakelov geometry to hyperbolic geometry, and then uses several methods available for the latter. a strategy turns out to be fruitful for many other research problems.

Besides his remarkable research results at the interplay between the theory of modular forms and Arakelov theory, Jürg Kramer is known to be an inspiring lecturer and a great teacher. His lectures and his discussions have provided students, colleagues, and collaborators with much inspiration and deep insights.

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Anna von Pippich got her doctorate under the supervision of Jürg Kramer from HU Berlin in 2010. She is currently a principal investigator in the LOEWE research unit "Uniformized Structures in Arithmetic and Geometry" at TU Darmstadt and GU Frankfurt.